from time to time upon the Moon. I may further illustrate the difficulty from the drawing now published in fig. 2. Much of the detail is evidently drawn with the greatest care, and yet the large ring shown to the north of *Cassini* does not exist upon the Moon. Careful study will show how it was suggested ; but at the same time it will also show that it was only suggested, and that a part of what is there drawn has no real existence.

I cannot conclude this short note without endeavouring to express a sense of the debt which all who are interested in this question must feel that they owe to Dr. Rambaut and the staff of the Radcliffe Observatory for the care with which they have gone into the matter and the trouble they have taken, including as it did the learning of the system of shorthand in which Russell made the only notes that we have in explanation of the drawings. That the result has not been more definite is due to the inherent difficulty of the inquiry.

On the Relative Efficiency of Different Methods of Determining Longitudes on Jupiter. By A. Stanley Williams.

1. Two chief methods have been employed for the purpose of determining the longitudes or positions of markings on *Jupiter*, and therefrom the rotation period of the planet. One of these consists in recording the times when the markings appear to be exactly in mid-transit across the disc of the planet. This is termed *the method of transits*. In the other method the positions of the markings are measured with a micrometer from the preceding and following limbs of the planet, and the time of transit derived from these measures. This is termed *the micrometric method.*

2. As is well known, very great claims have been made as to the superiority of the micrometric method over the method of transits as regards the accuracy of the results obtained. Thus, the former has been described as being "infinitely preferable" to the latter ; and positions determined by the method of transits have even been said to bear the same relation to those derived from the micrometric method, as eye-estimates of the position angles and distance of double stars possess with respect to micrometer measures of the same. As the number of positions already determined by the method of transits can hardly amount to less than 40,000, and as these constitute the foundation of much of our knowledge of the rotation of *Jupiter* and of the various surface currents known to exist, it seems desirable to make some inquiry into the relative efficiency of the two methods.

3. Reference should be made in this connection to some statements and comparisons on this subject by Professor G. W. Hough in *Astronomy and Astro-physics*, vol. xi. p. 193, and in *Popular*

Astronomy, 1903, p. 297. A few words by Professor E. E. Barnard will also be found in *Publications of the Astronomical Society of the Pacific*, vol. i. 91, and in the *Monthly Notices*, vol. lii. 11. The writer may likewise, perhaps, be permitted to refer here to a few remarks on the subject by himself published in *Popular Astronomy*, 1903, p. 188. But, somewhat singularly, in view of the strong claims that have been made as to the superiority of the micrometric method, no adequate investigation or comparison as to the relative accuracy of the results obtained by the two methods seems to have been hitherto undertaken.

4. In the "Report of the Director of the Dearborn Observatory" for 1882 it is stated that the mean error on the concluded time of transit from a single pair of measures was $\pm 0{\cdot}9$ minute, and for the "average mean probable error" for any day was $\pm 0{\cdot}4$ minute. But these values were based on observations of the red spot, when at its greatest plainness, made on thirty-one nights only; and they seem to have been derived from the accordance of the separate measures of each night *inter se*. Observers of double stars are well aware that the separate measures of a star made on the same night may be very accordant, although the mean results of several different nights may differ considerably, and for this reason it is usually considered preferable to make a small number of measures of a star on each of several different nights rather than a large number on a single night. But there are other reasons for expecting a similar but much larger difference in the case of measures of planetary markings. Nevertheless, in whatever manner derived, it is from the comparison of the above small mean or probable errors with those derived by comparing positions obtained by the method of transits on *different* nights that the supposed superiority of the micrometric method would seem to have been founded.*

5. In order to make any proper comparison between the two methods it is obviously essential to choose some mode that shall be perfectly fair and do equal justice to both. Now since an observer can only observe a transit of a spot once in the course of the same passage of the marking across the planet's disc, it is clear that it is impossible to directly derive the mean or probable error of such transit on any particular night from the observations of that night. Hence the only simple mode of comparison that appears to be suitable and yet perfectly fair to *both* methods is to compare all the observations of a spot with an ephemeris representing its motion, and thence to deduce the mean or probable error of a night's work for that particular spot. The mean result derived in this manner from a number of spots will then give a good idea of the real degree of accuracy attained by either method.

* The actual mean error of a night's work on the red spot, according to the eighty-two observations published in the above cited report, is $\pm 1^{\mathrm{M}}{\cdot}5$, or nearly four times greater than the figure stated above for the average mean probable error.

6. It is obviously essential not to rest our conclusions on the results obtained from one or two spots only and a limited number of observations, but to discuss a considerable amount of work accomplished by both methods in order to satisfactorily compare the two. The present paper therefore includes in the first place a discussion of the micrometrical work published by Hough in the *Monthly Notices*, vol. lx. p. 546 *et seq.* This should give a very fair idea of the general accuracy of the micrometrical method, as a good many spots were observed, and the observations of each spot are usually fairly numerous. Moreover, the very clear manner in which the work is presented renders the discussion of the results easy and pleasant. In order to render the comparison more complete, and to avoid doing any apparent injustice to the micrometric method, the fine series of observations of the red spot, eighty-two in number, made in the years 1879–82 has been included in the discussion.

7. We have in the first place, besides the results of 1879–82, four years' observations of the red spot. The first column below gives the year or years in which the observations were made, the second column the *mean error* * of an observation or night's work, and the third the number of observations from which this has been derived. Then follow similar particulars for the other spots observed, in the order in which they come in the *Monthly Notices*, with an additional column descriptive of the spots. The work on the red spot has been considered separately from that done on the other spots.

The Red Spot.

	Year.	Mean Error.	Observations.
		M	
	1879–82	1·5	82
	1895–6	1·7	23
	1897	2·0	15
	1897–8	1·7	23
	1898–9	1·6	27
Average mean error for red spot =		± 1·7	170

Other Spots.

Black spot B_3	1895–6	1·6	7
"	1897	2·2	9
"	1898	0·8	7
Black spot *a*	1895–6	2·2	16
Black spot *b*	1895–6	3·6	22
Black spot *a*	1898	2·7	18
Black spot *b*	1898	2·1	16

* Airy's "Theory of Errors of Observations," articles 26 and 61.

	Year.	Mean Error. M	Observations.
Black spot	1897	0·8	4
Black spot	1897	0·6	4
Black spot	1897	1·4	4
Black spot	1897	3·0	6
Black spot C	1898	1·5	7
Long black spot	1899	3·9	5
White spot	1899	1·9	14
White spot	1899	2·7	7
Average mean error for other spots = ± 2·1			146

8. It should be mentioned that three spots have not been included in the discussion, owing to their having only been observed on three nights each, and this number is too small to enable the mean error to be derived with any certainty, besides which there is necessarily usually some doubt as to identification, where there are only three observations of a spot. The first white equatorial spot referred to on p. 555 of the *Monthly Notices* has also been excluded, as the identification does not seem to the writer to be correct, the four observations given appearing to relate to three different spots. Moreover, the equatorial spots are seldom suitable for our present purpose on account of the great and rapid changes of position and appearance to which they are subject.

9. It will be seen from the foregoing table that the average mean error of an observation or night's work is $\pm 1^{M}\cdot 7$ in the concluded time of transit of the red spot, and $\pm 2^{M}\cdot 1$ in the case of other spots. With regard to the latter there are considerable variations for different spots, as might have been anticipated, for spots presenting the size and uniformity of shape and aspect of the red spot are rare. These figures differ very considerably from the $\pm 0^{M}\cdot 4$ given by Hough as the probable error of a night's work. But they are the only ones at all comparable with what has been hitherto published regarding the errors of the method of transits. Even when the red spot was at its maximum plainness (1879–82) the mean error of an observation by the micrometric method was only reduced to $\pm 1^{M}\cdot 5$; a figure which, rather curiously, is exactly the same as that derived from ninety-seven contemporary observations of Schmidt made by the method of transits, as will be seen later on. It may therefore be concluded that the times of transit of spots on *Jupiter* derived by the micrometric method are subject to an average mean error of $\pm 1^{M}\cdot 7$ in the case of the red spot and $\pm 2^{M}\cdot 1$ for other spots.

10. In order to fairly compare the results obtained by the two methods it will be necessary to determine the mean error of an observation, or night's work, in the same manner as has now been done for the micrometric method, for a number of spots observed by the method of transits. Only comparatively few of

the numerous observations made according to this method have been published in the very clear and detailed manner that has been employed by Hough, so that in the great majority of cases a considerable amount of labour and time would necessarily have to be devoted to the purpose in order to extract the necessary information. Nevertheless sufficient material is already to hand to enable the mean error to which the method of transits is subject to be determined in a manner comparable with that done for the micrometric method.

11. The results contained in the following table have been extracted from various sources, and are based on the work of several observers. The list is not by any means supposed to be exhaustive, but, as every instance found by the writer and published in sufficient detail has been made use of, it is probable that they are fairly representative of the method. No attempt has been made to select cases in which the observations are specially accordant or to reject any that are unusually discordant. The only cases excluded are two or three instances of spots of which only three or four observations are available, and where consequently a satisfactory value of the mean error cannot be determined. The table contains the same particulars as the previous one, with the addition in the last column of the name of the observer and a reference to the source from which the result has been derived.

The Red Spot.

Year.	Mean Error. M	Observations.	Observer.
1880–2	1·0‡	44	Barnard (*Pub. A.S.P.*, i. 91)
1891	0·8*	11	„ (*Monthly Notices*, lii. 12)
1898	2·2†	10	Denning (*ibid.*, lix. 82)
1899	1·3†	35	„ (*ibid.*, lx. 217)
1898	1·5†	14	Gledhill (*ibid.*, lix. 82)
1899	1·0†	23	„ (*ibid.*, lx. 217)
1880–1	0·5	11	Jedrzejewicz (*A.N.*, 2366)
1898	1·4†	8	Phillips (*Monthly Notices*, lix. 82)
1899	1·6†	26	„ (*ibid.*, lx. 217)

‡ Barnard states that "among the observations of the red spot I have forty-four complete and carefully estimated transits—that is, observations of the preceding end, middle, and following end of the spot. Twenty-one of these are from a single but careful estimate of each phase. These give the probable error of a transit of the centre from the mean of the three observations $= \pm 1^{m}\cdot 0$. In twenty-three of these transits three estimations were made of each phase: from these I get for the transit of the middle from the mean of the nine observations the error of the transit, $= \pm 0^{m}\cdot 7$" (*Pub. A.S.P.* vol. i. p. 91). The mean error of a transit for 1880–82, the period in which these 44 observations were made, has been assumed to be $\pm 1^{m}\cdot 0$ in the table. In all other cases the mean error has been derived directly by the writer.

† In these cases the "hollow" or bay in the S. equatorial belt was observed, and not the actual spot. Owing to the faintness of the latter all observers except the writer seem during the last few years to have relinquished observing the spot in favour of the "hollow."

Year.	Mean Error. M	Observations.	Observer.
1879–81	1·5	97	Schmidt (*A.N.*, 2410)
1892	2·0*	25	Williams (*Mem. B.A.A.*, ii. 143)
1893–4	2·6*	14	„ (*ibid.*, iii. 138)
1898	2·0	15	„ (*Monthly Notices*, lix. 82)
1899	3·1	9	„ (*A.N.*, 3596)
1900	0·8	4	„ (*ibid.*, 3675)
1901	2·3	14	„ (*ibid.*, 3786)
1902	2·2	22	„ (*ibid.*, 3875)
Average mean error	= ±1 6	382	

Other Spots.

	Year.	Mean Error. M	Obser-vations.	Observer.
Dark spot ...	1895–6	2·4	13	Brenner (*Monthly Notices*, lvi. 531)
White spot I...	1898	2·9	6	Denning (*ibid.*, lix. 85)
Dark ellipse ...	1898	2·7	5	„ (*ibid.*, lix. 87)
Dark spot II....	1898	(7·5)	6	„ (*ibid.*, lix. 88)
Dark spot ...	1895–6	2·0	30	Gledhill (*ibid.*, lvi. 531)
Dark spot ...	1895–6	2·4	5	Merfield (*ibid.*, lvi. 531)
Dark ellipse ...	1898	(5·8)	8	Phillips (*ibid.*, lix. 87)
Dark spot ...	1898	2·1	9	„ (*ibid.*, lix. 88)
Dark spot ...	1895–6	0·7†	7	Rambaut (*ibid.*, lvi. 531)
Dark spot ...	1862	0·6	12	Schmidt (*A.N.*, 2342)
Sharp elbow (Ecke)	1862	3 0	6	„ (*ibid.*, 1973)
White spot ...	1866	1·2	5	„ (*ibid.*, 1973)
White spot ...	1890	2·6*	20	Williams (*Monthly Notices*, lviii. 323)
End of belt ...	1881	1·2	9	„ (*ibid.*, lix. 29)
Dark spot ...	1888	2·6	10	„ (*ibid.*, lix. 30)
White spot II.	1898	1·0	5	„ (*ibid.*, lix. 84)
White spot I...	1898	2·7	5	„ (*ibid.*, lix. 85)
White spot III.	1898	0·6	5	„ (*ibid.*, lix. 85)
Dark spot ...	1892	1·7	16	„ (*A.N.*, 3528)
Dark spot ...	1890	2·5	8	„ (*ibid.*, 3528)
Average mean error		= ±1·9	176	

12. There are some circumstances affecting the results contained in the foregoing table that do not favour the method of

† It is assumed that Rambaut's observations were made by the method of transits, but the writer has not seen the original paper from which they were taken.

transits. In the cases marked with an asterisk the motion of the spot was assumed to conform to Marth's ephemeris. This, however, was probably never exactly so, and consequently the stated values of the mean error are no doubt somewhat too great. Also in several cases the observations of a spot were compared with an ephemeris representing its mean motion as derived from the work of several observers, and a discussion based solely on the observations of the particular observer would probably result in reducing the residuals, and so the mean error. It may therefore be safely assumed that the average mean errors derived from the table are, if anything, too large. Schmidt's fine observations of the red spot are corrected for his "Correction C," representing a constant error varying according to the hour-angle of *Jupiter* at the time of observation. Schmidt seems to have connected this constant error with the inclination of the belts to the line joining the two eyes when the planet is far east or west of the meridian, and the head is held level. Probably he was right in this, as no similar error seems to be revealed by the work of other observers, most of whom, it is believed, take the precaution of keeping the eyes parallel to the belts when observing the transits. The two bracketed results in the list of "other spots" have been omitted in forming the average, as they are largely in excess of any others, and relate to two faint and difficult spots in a rather high S. latitude.

13. We are now in a position to make a proper comparison of the results obtained by the two methods. We have :—

Red Spot.

	Average Mean Error. M	No. of Series or Spots.	No. of Obs.
Micrometric Method ...	$= \pm 1{\cdot}7$	5	170
Method of Transits ...	$= \pm 1{\cdot}6$	17	382
	Other Spots.		
Micrometric Method ...	$= \pm 2{\cdot}1$	15	146
Method of Transits ...	$= \pm 1{\cdot}9$	18	176

It will be seen that, so far from the micrometric method showing any superiority in point of accuracy over the method of transits, the advantage is rather in favour of the latter. The difference is, however, only slight, and it may therefore be concluded that the two methods yield results of about equal accuracy.

14. By largely increasing the number of measures of a spot theoretically* greater accuracy could be secured by the micrometric method than by the method of transits, though it would

* Practically it seems doubtful, however, whether any real increase in accuracy would result, on account of liability to bias in the later measures. Some years ago, when measuring some photographs of *Jupiter*, the writer found this liability to bias so marked as to render it expedient not to measure the different images of the planet in order of time; but this cannot be managed in the case of direct measures on the planet at the telescope.

seem that the beneficial effect of this would be slight. For the mean error is no doubt made up chiefly of three quantities: (*a*) the real errors of observation; (*b*) apparent errors due to the motion of the spot not being perfectly uniform, and (*c*) apparent errors produced by changes in the form and appearance of the spot. The apparent errors *b* and *c* will balance themselves when a large number of observations and many spots are discussed, so that their effect may be considered as the same for both methods. Now if $\pm 0^{M}\cdot 4$ be taken to represent the real error, then doubling the accuracy of the observations would only result in reducing the mean error from, say, $2^{M}\cdot 0$ to $1^{M}\cdot 8$. Considering the nature of the problems requiring to be investigated, it seems doubtful whether the great increase in the labour and time necessitated by the larger number of measures would be compensated for by the slight benefit obtained.

15. It has been said that an eye-estimate of a transit is, in a manner, equivalent to a single micrometer measure, and liable to the same error of bisection, &c.; but this, I think, is clearly not so. In a complete single measure, as described by Hough, one micrometer wire is placed tangent to the preceding limb of the planet, and the other made to bisect the spot. Both wires are then carried across the disc, and one placed tangent to the following limb and the other made to bisect the spot. It would seem, therefore, that in every complete measure there are four distinct sources of error. There is the liability to error in twice placing the wires tangent to the two limbs, and there is the liability to error in twice bisecting the spot. In the method of transits the comparisons are made direct with the true limbs, so that the liability to error in placing the wires tangent to the latter does not arise. Then the spot itself takes the place of the other wire, and there is consequently only a single bisection here—the bisection of the disc. Hence there are four distinct sources of error to which a single measure by the micrometric method is liable, compared with one in the method of transits. The effect produced by the rapid rotational motion of the spots must also not be lost sight of in making comparisons of this kind. If the rotation of *Jupiter* were performed in one hour, instead of nearly ten, the advantage would be all on the side of the method of transits.

16. It is not proposed to consider here the question of personal equation. It is only of importance where contemporary observations of the same marking by two or more observers are combined, but there is no reason for supposing that it would be any less in the case of the micrometrical method than it is known to be where the method of transits is used. It is no doubt chiefly due to different observers not observing markings, irregular both in shape and intensity, and often very highly so, in exactly the same way.

17. The conclusions derived from the foregoing investigation may be shortly stated as follows;

(1) The alleged great or indeed any superiority in accuracy of the micrometric method over the method of transits does not exist.

(2) The two methods give results of about equal accuracy.

(3) The times of transit derived from either method are subject to an average mean error of $\pm 2^{M}\cdot 0$, but in the case of very prominent and definite spots, such as the red spot in the years 1879–82, the mean error may be reduced to $\pm 1^{M}\cdot 5$ or even less. On the other hand, it often largely exceeds $2^{M}\cdot 0$, especially in the case of faint or irregular markings.

18. As the rotation of *Saturn* is performed in nearly the same time as that of *Jupiter*, it is evident that the first two conclusions apply with equal force to the former planet; so that the statement made in the *Monthly Notices*, vol. lxiv. 122, as to the great superiority of the micrometric method for determining the positions of spots on *Saturn* does not appear to be justified.

Hove: 1904 *February* 12.

Positions and Photographic Magnitudes of ninety Stars surrounding the Variable R Cygni. By Joel H. Metcalf, Ph.D.

(*Communicated by Professor H. H. Turner.*)

1. The suggestion that such work as the following might have value came from the recent experience of the Oxford Observatory in the work on the Rousdon variable stars. Existing positions for the comparison stars are usually given only roughly, and sometimes there is a doubt about identification in consequence. It was therefore felt that accurate places would be desirable. In this particular case it will be seen that the places good enough for a working chart at Rousdon were sometimes 5′ in error.

2. There would also seem to be good general reasons for accurate surveys of special regions, especially those that were previously known to contain objects of interest. On this account the region of variables is sure to be under close observation, and a detailed study of them will be given which it will not be possible to extend to the whole sky.

3. The photographic plate, which is the basis of the following catalogue, was taken at the University Observatory with the 13-inch astrographic telescope on the night of 1903 November 15. The plate was exposed from $1^{h}\ 8^{m}\ 50^{s}\cdot 0$ to $2^{h}\ 9^{m}$ local sidereal time. As a few seconds of time were lost in the middle of the exposure the time was approximately one hour. The plate is a good one and shows stars of about the 14th magnitude, as one